Quiz 9

Monday, June 13, 2016 1:02 PM

Use Stokes' Thm to compute $\iint_S \nabla \times \vec{F} \cdot d\vec{S}$ where $\vec{F} = -y\hat{i} + x\hat{j} - 2\hat{k}$,

where S is the cone $z^2 = x^2 + y^2$, $0 \le z \le 4$, oriented downward.

<u>Soln</u> The boundary curve C is the circle $x^2+y^2 = 16$, $z = 4$ oriented in the clockwise direction as viewed from above (as S is oriented downward)

Parametrize C by $\vec{r}(t) = 4\cos t\,\hat{i} - 4\sin t\,\hat{j} + 4\hat{k}$, $0 \le t \le 2\pi$

as it is oriented clockwise

Then $\vec{r}'(t) = -4\sin t\,\hat{i} - 4\cos t\,\hat{j}$

$\vec{F}(\vec{r}(t)) = 4\sin t\,\hat{i} + 4\cos t\,\hat{j} - 2\hat{k}$

Then by Stokes' Thm,

$$\iint_S \nabla \times \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\,dt = \int_0^{2\pi} \langle 4\sin t, 4\cos t, 4 \rangle \cdot \langle -4\sin t, -4\cos t, 0 \rangle\,dt$$

$$= \int_0^{2\pi} -16(\cos^2 t + \sin^2 t)\,dt = \int_0^{2\pi} -16\,dt = -32\pi$$